EVOLUTION OF NEUTRAL AND CHARGED DROPLETS IN AN ELECTRIC FIELD

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Summary We study the evolution of drops of a very viscous and conducting fluid under the influence of an external electric field. The drops may be neutral or may be charged with some amount of electric charge. If both the external electric field and total drop charge are sufficiently small, then prolate spherical shapes develop according to Taylor's observations. For sufficiently large charge and/or external field a selfsimilar cone-like singularity develops in a mechanism different from Taylor's prediction. The opening semiangle of the cones both for uncharged and charged drops in a constant electric field is typically around 30^0.

INTRODUCTION

An important problem in fluids dynamics is the formation of singularities on charged masses of fluid. This problem is relevant in a variety of physical and technological situations, such as the breakup of water droplets in thunderstorms, electrospraying, electrospinning, an electropainting. Here we focus our attention in the evolution and breakup of drops of a conducting fluid, both neutral and electrically charged, under the influence of an external electric field. This problem has been the subject of numerous studies since the pioneering works of Rayleigh, Zeleny and Wilson & Taylor ([6], [7], [8], [9]), where stability of spherical or spheroidal drops was analyzed and singular fluid interfaces in the form of cones were observed and discussed.

On the other hand, the development of experimental settings able to register the very fast events previous to the disintegration of charged drops in an electric field ([3], [4]) has shown that such disintegration takes place after the development of conical tips from which ultrathin jets are emitted. In the case of isolated charged drops of a conducting fluid it was shown ([1]) that the evolution leads to the formation of a cone-like singularity that develops in a selfsimilar manner. An important characteristic of such cones is that they have an opening semiangle much smaller that the classical stationary Taylor's cone. They result from a balance of viscous stresses and electrostatic forces instead of the static balance between capillary an electrostatic forces.

We show that cones of the type described above also develop when charged drops are immersed in a constant external electric field, such as the one created in a planar capacitor. Interestingly, the opening semiangle is essentially independent of both the external field and the net charge in the drop. When drops are charged, the opening semiangle is different but still independent of the value of the imposed field. Our investigation relies in the use of a boundary element method combined with a asymptotic analysis of the behaviour of both the flow and the electric field near the singularities..

THE MODEL EQUATIONS

The drop occupies a region $\Omega(t)$ and the liquid of the drop is a perfect conductor with infinite conductivity. Hence the electric potential V is constant inside and at the drop surface, and all the electric charge will be located at the boundary. Since the surrounding medium is a dielectric, the total charge Q remains constant. The electric field $\mathbf{E}$ outside the drop is given by $\mathbf{E} = -\nabla V$ where V is an harmonic function with constant value at the surface of $\Omega(t)$. The constant potential C has to be chosen such the total charge is Q. At the surface of a conductor. the surface charge density σ is then given by the normal derivative of the potential, $\sigma = -\varepsilon_0 \frac{\partial V}{\partial n}$, so that the repulsive electrostatic force per unit area is $\frac{\sigma^2}{2\varepsilon_0}\mathbf{n}$ where $\mathbf{n}$ is the outward normal to the surface.

The fluid velocity $\mathbf{u}$ and the fluid pressure p inside the drop satisfy the Stokes equations

$$-\nabla p + \mu_1 \Delta \mathbf{u} = 0, \ \nabla \cdot \mathbf{u} = 0 \ \text{ in } \Omega(t) \tag{1}$$

where μ_1 is the viscosity of the liquid inside the drop. Equations similar to (1) must be satisfied by the velocity and the pressure outside of the drop with μ_1 replaced by μ_2, the viscosity of the surrounding liquid. The boundary condition for the stress is

$$(T^{(2)} - T^{(1)})\mathbf{n} = \left(\gamma\kappa - \frac{\sigma^2}{2\varepsilon_0}\right)\mathbf{n} \quad \text{on } \partial\Omega(t), \tag{2}$$

where κ is the mean curvature of the surface, i.e. the average of principal curvatures, and $T^{(k)}$ is the stress tensor inside ($k = 1$) or outside ($k = 2$) the drop. Equation (2) expresses the balance between viscous stress, capillary forces and electrostatic repulsion. The kinematic condition is $v_N = \mathbf{u} \cdot \mathbf{n}$ on $\partial\Omega(t)$ where v_N is the normal velocity of the free boundary $\partial\Omega(t)$.

The problem can be easily recasted in terms of three dimensionless parameters: 1) the viscosity ratio $\lambda = \mu_1/\mu_2$, 2) Rayleigh's fissibility ratio $X = \frac{Q^2}{24\gamma\pi\varepsilon_0 Vol.}$,where $Vol.$ is the volume occupied by the drop, and 3) the reduced external electric field $E_\infty = \sqrt{\frac{\varepsilon_0 Vol^{\frac{1}{3}}}{\gamma}}\mathcal{E}_\infty$ .Our results will be expressed in terms of these dimensionless parameters.We have implemented a boundary integral method in order to solve both the equations for $\mathbf{u}$ and V.

RESULTS AND CONCLUSIONS

We have simulated the time evolution of a drop that initially is a prolate spheroid ($a = c = 0.6, b = 0.66$). The inner fluid has a viscosity of $\mu_1 = 1$ and the outer fluid a viscosity of $\mu_2 = 0.4$. We have considered two cases, the first one deals with an uncharged drop ($Q = 0$) under an electric field $E_\infty = 0.436$ and the second one considers a charged drop $Q = 0.24Q_c$, where Q_c is the critical Rayleigh limit (see [6]) and an electric field $E_\infty = 0.4$. The results are depicted in figure 1.

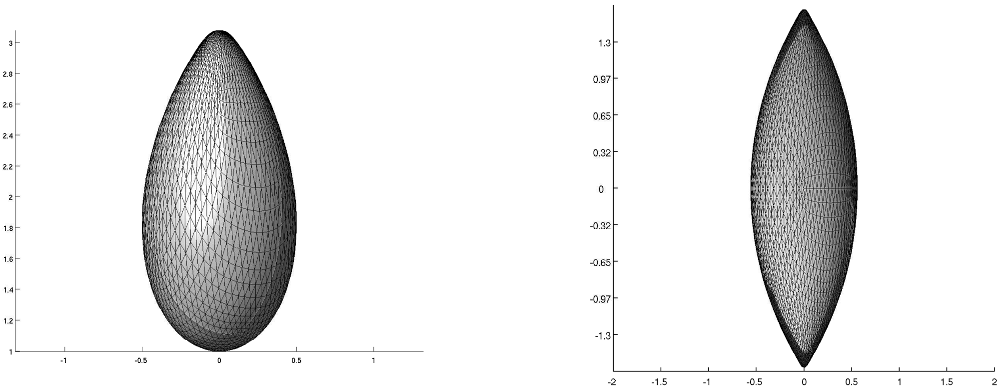

Figure 1. Shapes of the drops just before the formation of the singularity both in the charged and uncharged situations.

These results provide evidence of formation of finite time singularities in both cases but with different shapes. In the case without charge there is a cone-like singularity that appears in both tips of the drop. This result matches the experimental result obtained in [5] just before the drop emits a thin jet from both tips. In the second case, where the drop is charged before the application of the electric field there is only one singularity in one side of the drop. In this case, again, the numerical result coincides with the experimental result obtained in [5].
When $X = 0$ the theoretical work by Taylor [7] showed that under a certain critical value of the electric field stationary spheroids should exist and established that their shape should be accurately approximated by ellipsoids. Above the critical value for E_∞, spheroids evolve towards the formation of cones at the tips as we have seen in the previous results. We have reproduced both cases numerically and we have obtained numerically Taylor's limit ($E_\infty \approx 0.412$). When $X > 0$ and for E_∞ above $E_c(X)$ (Taylor's limit when $X > 0$) drops deform into ovoids. For a given value of X ($0 < X < 1$) there is a critical value $E_c(X)$ of the external electric field E_∞ such that there exist stable ovoids, drifted by the electric field, whenever $E_\infty < E_c(X)$. We have computed numerically the curve $E_c(X)$ and various parameters such as aspect ratios and the angle of the conical singularities as a function of the parameters of the problem..

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